ABSTRACT

This paper deals with the transition between different primary visual servoing tasks, which may induce undesired behavior if the switch is not smooth. Indeed, possible transition functions to switch from a first primary task to a second one are given. In addition, an intermediate stopping-phase strategy with a proper parameterization in the image plane is also demonstrated. Experimental results for a switching between a hallway following task and a positioning task are shown to validate the approach.

KEYWORDS: visual servoing, switching, vision-based control, robotics.

1 INTRODUCTION

Complex robotic tasks may be combined as a set of several primary actions. To completely perform the whole assignment, a decomposition into elementary tasks may be applied. However, undesired behavior (or even system failure) may arise during the transient phase, as well as the system stability and robustness are not ensured, if the switching between the elementary tasks is not well formulated. In order to avoid such hypothesis, a smooth transition or an intermediate stopping phase strategy deployment is claimed.

In this work, besides the proposition of smooth switching control laws instead of working on the image errors as in Pissard-Gibollet (1993), experimental and simulation results are also demonstrated validating the strategies. Furthermore, the parameterization of the intermediate stopping pose in made directly in the image plane instead of a 3D pose. The experimental platform is shown in Fig. 1, and it consists of a synchro-drive mobile robot with a camera mounted on a pan-tilt unit placed on its top. An image-based visual servoing approach (Espiau et al., 1992) is extensively used at the vision-based control level for stability and robustness reasons.

The article is organized as follows. Section 2 briefly reviews some important aspects of visual servoing, where a sufficient condition for stability is presented along with the general framework to deal with any 2D visual servoing tasks. Section 3 introduces two strategies to smoothly switch between two generic visual tasks. The 2D tasks considered as examples are then modeled in Section 4, which are the hallway following and the positioning tasks. Section 5 presents several results of the switching and Section 6 concludes the paper and points out future works.

2 VISUAL SERVOING

Visual servoing has the aim to control the robot in closed-loop through the use of image feedback. Imagery information acquired by a camera mounted on the robot can be used to control directly its motions, a method called image-based visual servoing (or 2D), or the imagery information can be used to recover the robot 3D pose to allow the deployment of well-established robot control strategies. Main difficulties of the latter consist of high processing power requirement, need for an adequate robot and camera calibration, and for the object reconstruction (refer to Silveira (2003) for comparative results).

Figure 1: The robot Nomad 200 used in the experiments.
2.1 Preliminaries

Let a robot frame $F$ whose pose $\xi \in \mathbb{R}^m$ is the vector containing the global coordinates of an open subset $\mathcal{X} \subset \mathbb{R}^1 \times SO(3)$, and $\xi_c$ is the pose of the camera frame $F_c$ w.r.t. $F$. Consider an appropriate image feature extraction function $\psi : \mathcal{Z} \subset \mathbb{R}^2 \rightarrow \mathbb{R}^l$ to construct a vector of image parameters $s = \psi(\xi_c, t)$. In this study, the authors focused on 2D visual servoing, and it is based on an interaction matrix $L = L(s, Z) \in \mathbb{R}^{l \times m}$ between the motion of $s$ and the robot motions $\xi(t) = [v(t), \omega(t)]^T \in \mathbb{R}^m$, which may be derived from the chain rule

$$\frac{ds}{dt} = \frac{\partial s}{\partial \xi_c} \frac{\partial \xi_c}{dt} + \frac{\partial s}{\partial \omega} \frac{\partial \omega}{dt}, \quad (1)$$

what allows to write in classical form as

$$\dot{s}(\xi_c, t) = L(s, Z)J_s(\xi_c)\dot{\xi}(t) + \frac{\partial s}{\partial \omega} \frac{\partial \omega}{dt}, \quad (2)$$

where $l \in \mathbb{N}$ is the minimal parameterization of $p \in \mathbb{N}$ image features, $m \in \mathbb{N}$ is the d.o.f. of the robot, $Z \in \mathbb{R}$ is the distance between a frame attached to the object $F_o$ and $F_c$, $J_s(\xi_c)$ is the robot Jacobian, and $\partial s/\partial \omega$ represents the change of image parameters due to possible target motion.

The control objective in 2D visual servoing is to perform camera motions such that the current visual signals approaches the desired ones $s(\xi_c, t) \rightarrow s_d$, $\forall t \in [0, T]$. Consider an error control vector as follows

$$e(\xi, t) = C(t)(s(\xi_c, t) - s_d), \quad (3)$$

where $C(t)$ is a combination matrix which also permits to include redundant signals.

**Lemma 1** Let us assume a combination matrix $C = (LJ_r)^\dagger$, where the overscript $(\cdot)^\dagger$ denotes the Moore-Penrose pseudo-inverse. Sufficient condition for the existence of Eq. (3) are such that if $||s - s_d||$ is sufficiently small then $e = 0$ only if $s = s_d$ to assure that $s - s_d$ never belongs to $\mathcal{N}(C(t))$.

From the time derivative $\dot{e}(\xi, t) = \frac{\partial}{\partial \xi_c} \frac{\partial s}{\partial \xi_c} + \frac{\partial}{\partial \omega}$, it is possible to derive

$$\dot{e}(\xi, t) = \dot{e}(\xi, t) - \frac{\partial e}{\partial t}, \quad (4)$$

if $\frac{\partial e}{\partial s} = 1 |_m$ (see Espiau et al. (1992) for necessary and sufficient conditions), with $\frac{\partial e}{\partial t}$ being an estimation since it represents the contribution of the target’s own motion.

Using a first order approximation $\dot{e} = -\lambda e$ along with Eq. (3), a simple proportional velocity controller $u(t) = \xi(t)$ leads to the control law

$$u(t) = -\lambda(LJ_r)^\dagger [(s - s_d) + o(||s - s_d||)] - \frac{\partial e}{\partial t}, \quad (5)$$

where $o(||s - s_d||)$ is the higher order error term, and with a constant gain $\lambda > 0$.

2.2 Stability analysis

In order to focus on the transition between tasks, let us assume a motionless target, that is $\frac{\partial \omega}{\partial s} = \frac{\partial \omega}{\partial t} = \frac{\partial \omega}{\partial \omega} = 0$.

**Theorem 2** Using the results from Lemma 1, the necessary and sufficient condition for the asymptotic stability of the equilibrium point $e(\xi, t) = 0$ is

$$(\hat{\lambda}J_r)^\dagger L_{J_r} > 0,$$

where a constant model $\hat{L} = L(s_d, Z_d)$ is usually used.

**Proof:** Substituting the control law (5), $u = \dot{\xi}(t) = -\lambda e$, in Eq. (2) along with the time derivative of (3), it is possible to achieve the closed-loop equation

$$\dot{e} = -\lambda(\hat{\lambda}J_r)^\dagger L_{J_r}e.$$

Then, Eq. (7) assures $||e(\xi, t)|| \rightarrow 0$ as $t \rightarrow \infty$ if the positivity condition of Eq. (6) is satisfied, i.e. $(\hat{\lambda}J_r)^\dagger L_{J_r}$ has eigenvalues with positive real part.

The model $\hat{L} = L(s_d, Z_d)$ is frequently used in practice in order to avoid the computation of the pseudo-inverse as well as $Z$ at each iteration.

**Corollary 3** The linearization $\hat{L}$ around $s = s_d$ ensures Theorem 2 if the starting point $e(\xi_0, t_0) = e_0$ is within a ball of attraction $B_r(e = 0)$, whose radius $r > 0$ (quite difficult to obtain analytically) is independent of $t_0$. Therefore, the equilibrium point $e(\xi, t) = 0$ is only locally asymptotically stable.

2.3 The general framework

Depending on the task, there may be $c = m - rank(L)$ d.o.f.s. unconstrained by the visual servo control. A general framework to gather the primary sensor-based task $e_p, (3)$ and a secondary $c$-dimensional objective $e_s$ to define a global vector of a task $i$ as $e_i = [e_p^T, e_s^T]^T$ such that those tasks must be compatible and independent, i.e.,

$$e_i = W^+ e_{p_i} + \alpha_e W^+ e_{s_i}, \quad (8)$$

where $\alpha_e > 0$ is a weighting scalar, $W$ is a full-rank coupling matrix such that $R(W^T) = R(L)$, $(\cdot)^\dagger$ is the orthogonal projection operator on $\mathcal{N}(L)$, that is, $W^\perp = \mathbb{I}_m - W^\dagger W$ in order to $W^\perp W^\dagger e_{s_i} = 0, \forall e_{s_i}$.

**Lemma 4** Therefore, Eq. (5) is a particular control law used when $c = 0$, that is, when the visual servoing primary task has a rigid virtual linkage. This general framework permits to relax Lemma 1 by setting $C = W(LJ_r)^\dagger$. 
3 SWITCHING STRATEGIES

Frequently in robotic systems, there are conflicting objectives that have to be properly dealt. For example, mobile robots are commanded to navigate within their workspace (task $e_1$) under obstacle avoidance behavior (task $e_2$). In such case, a minimization procedure of a cost function

$$\min_e J = \frac{1}{2} \| e_1^T Q_1 e_1 + e_2^T Q_2 e_2 \|^2_2,$$  \hspace{1cm} (9)

with weighting matrices $Q_1, Q_2 > 0$, could be deployed to manage those tasks.

The interest here is to smoothly commute from the visual servoing task $e_1$ to $e_2$ in order to improve the transient phases. Studies in this direction were conducted by Pissard-Gibollet (1993). Indeed, using Eq. (9) it is not appropriate when large changes in the task functions may be present (e.g., when a target appears suddenly) because, depending on $Q_1$ and $Q_2$, the robot actuators may be damaged.

3.1 Transition functions

Let $\mu \in [0, \tau]$ be an element of the switching period. Suppose it is desired to completely switch between two primary tasks $e_1(\xi, t)$ and $e_2(\xi, t)$ at $\mu = \tau > 0$. At a first glance, a transition function could be

$$\varphi(\mu) = \begin{cases} 1 & 0 \leq \mu < \tau \\ 0 & \mu \geq \tau \end{cases}.$$  \hspace{1cm} (10)

However, such a rough transition may cause serious damages to the robot mechanics due to $\lim_{\mu \to \tau} \xi(\mu) = \infty$, and also to the unpredictable results at the transient phase.

In order to smoothly transfer from $e_1$ to $e_2$, it is argued that a possibility is to define an odd function $\phi(\mu - \tau/2) : [0, \tau] \rightarrow [0, 1]$ around $\phi(\mu) = 1/2$ to balance equally between the two primary tasks, with $\phi(\mu) \in C^2$ for smoothness, $\lim_{\mu \to 0^-} \phi(\mu) = 1$, and $\lim_{\mu \to \tau^+} \phi(\mu) = 0$. Then, for the more general case ($\forall \mathcal{R}(L_1), \forall \mathcal{R}(L_2), \forall \mathcal{m}_1, \forall \mathcal{m}_2$), a switching error vector can be constructed as $e = [e_1^T(e_{p_1}, e_{s_1}), e_2^T(e_{p_2}, e_{s_2})]^T$, i.e.:

$$e = \phi(\mu)e_1 + (1 - \phi(\mu))e_2$$  \hspace{1cm} (11)

with

$$e_i = W_i^T e_{p_i} + \alpha_i(L_{m_i} - W_i^T W_i)e_{s_i}, \hspace{1cm} i = 1, 2,$$  \hspace{1cm} (12)

and $e_{p_i}$ comes from Eq. (3) along with the result from Lemma 4. Reminding that the $e_{s_i}$ will exist only if there are unconstrained d.o.f.s. left free from $e_{p_i}$, i.e. rank($L$) $< m$.

Possible transition functions to implement Eq. (11) are:

- a linear function:

$$\phi(\mu) = -\frac{1}{\tau} \mu + 1;$$  \hspace{1cm} (13)

- polynomial transition functions. Using the boundary conditions $\phi(0) = 1, \phi(0) = 0, \phi(\tau) = 0$ and $\phi(0) = 3$, that is, $\phi(\mu) = \alpha_3 \mu^3 + \alpha_2 \mu^2 + \alpha_1 \mu + \alpha_0$, with $\alpha_3, \alpha_2, \alpha_1, \alpha_0 \in \mathbb{R}$, which gives

$$\phi(\mu) = \frac{2\mu^3}{\tau^3} - \frac{3\mu^2}{\tau^2} + 1.$$  \hspace{1cm} (14)

\textbf{Corollary 5} By using such an approach, the definite positivity of the task Jacobian $\frac{\partial \dot{q}}{\partial \xi}$, along its own ideal trajectory can be assured through a proper choice of $e_i$. However, the definite positivity of the global Jacobian $\frac{\partial \dot{q}}{\partial \xi}$ can not be assured during the transient phase.

Then, it is possible to design a smoothly switching control law, instead of working with the global task error, as

$$u = \phi(\mu)u_{e_1} + (1 - \phi(\mu))u_{e_2}$$  \hspace{1cm} (15)

with $u_{e_i} = -\lambda_i e_i$ by using a simple proportional controller and the formalism introduced in Section 2.

3.2 Intermediate stopping phase

Another possibility is to consider a intermediate stopping phase at $\mu = 0$ before commuting to the second primary task $e_2$. Thus, Eq. (11) may be split into two that do not overlap, that is

$$e = [e_1^T(e_{p_1}, e_{s_1}), e_2^T(q, e_{s_2})]^T$$  \hspace{1cm} (17)

and

$$e_1 = \phi(\mu)e_1 + (1 - \phi(\mu))\dot{q}$$  \hspace{1cm} (18)

with $\dot{q} = q - q_d$ may have a desired position $q_d = q(\mu = 0)$ parameterized either in the image plane or as a 3D pose.

\textbf{Corollary 6} The intermediate stopping phase strategy guarantees stability if $\frac{\partial \dot{q}}{\partial \xi}$ is definite positive, since $\frac{\partial \dot{q}}{\partial \xi} > 0$ along its ideal trajectory. For $\dot{q}$ parameterized as a 3D pose leads to $\frac{\partial \dot{q}}{\partial \xi} = \begin{bmatrix} 1 & 0 & T \end{bmatrix}$, whose sufficient condition for its definite positivity is a well chosen attitude parameterization. For $\dot{q}$ parameterized in the image plane, see Theorem 2.

A smoothly switching control law with an intermediate stopping phase can then be

$$u' = \phi(\mu)u_{e_1} + (1 - \phi(\mu))u_{\dot{q}}$$  \hspace{1cm} (19)
4 PRIMARY TASKS MODELING

Several vision-based tasks can be performed robustly using visual servoing. Proceeding with the previous works done by the authors, where the hallway following (Shiroma et al., 2003) and the positioning (Silveira et al., 2001) visual servoing tasks were tackled, it is addressed here the issue of the proper switching between those tasks.

Corridor following tasks can be performed by ground mobile robots using only one line on the plane, since \( \xi_{G2}(t) = [v_x, \omega_y]^{T} = J_{r_1}[v, \theta]^{T} = J_{r_1}u_{v_1} \), where \( v \) and \( \theta \) are the translational and rotational velocities, respectively. Considering the polar representation for the line, it is possible to construct the visual signal vector as \( s_1 = [\theta, \rho]^{T} \), whose interaction matrix \( L_1 = [L_{\rho}, L_{\theta}]^{T} \) is given as (Espiau et al., 1992)

\[
\begin{align*}
L_{\rho} &= [ -\rho \cos \theta & \rho \sin \theta & -\rho & \cdots ] \\
L_{\theta} &= [ -\rho \cos \theta & \rho \sin \theta & -1 & \cdots ] \\
& \quad (1 + \rho^2) \sin \theta & -(1 + \rho^2) \cos \theta & 0 & \cdots 
\end{align*}
\]

(20)

where \( \lambda_{\rho} \) and \( \lambda_{\theta} \) can be computed from the equation of the ground plane in the camera frame. The \( L_1 \in \mathbb{R}^{2 \times 3} \) (because \( \text{dim}(SE(2)) = 3 \)) allows to control only \( \text{rank}(L_1) \leq 2 \) d.o.f.

For hallway following tasks, the d.o.f. left unconstrained is the position along the line, which requires a secondary task \( e_{s_1} \), to be designed.

Concerning the positioning task, which constrains all \( m \) d.o.f. of the robot, it can be performed through the use of \( p > 3 \) image points in order to obtain \( c = 0 \) (see Section 2.3), which allows us to write the visual signal vector as \( s_2 = [x_1, y_1, \ldots, x_p, y_p]^{T} \), whose well-known interaction matrix \( L_2 = [L_{x_1}^{T}, L_{y_1}^{T}, \ldots, L_{x_p}^{T}, L_{y_p}^{T}]^{T} \) using a pinhole camera model is given as

\[
\begin{align*}
L_{x_i} &= [ -\frac{1}{z_i} & 0 & \frac{x_i}{z_i^2} & x_i y_j & -(1 + x_i^2) & y_j ] \\
L_{y_i} &= [ 0 & -\frac{1}{z_i} & \frac{y_i}{z_i^2} & 1 + y_i^2 & -x_i y_j & -x_i ]
\end{align*}
\]

(21)

An approach to compute \( L \) analytically for generic geometric primitives can be found in Espiau et al. (1992), which is a consequence of the implicit function theorem.

For the case of a ground mobile robot performing positioning tasks, it can be shown that one more d.o.f. is needed (Tsakiris et al., 1998). Providing the camera pan motion as the additional d.o.f., \( \xi_{G2}(t) = [v_x, \omega_z, \omega_y]^{T} = J_{r_2}[v, \dot{\theta}, \dot{\theta}_{pan}]^{T} = J_{r_2}u_{v_2} \).

5 RESULTS

The results with the mobile robot shown in Fig. 1 are presented in this section to depict the behavior of the robot under the transition functions expressed in Eq. (10) (simulation only to avoid robot malfunctioning), and Eq. (14) for \( \tau = 5s \); as well as using the intermediate stopping phase methodology. For all strategies, the secondary task \( e_{s_1} \) was a simple constant forward speed of at maximum 8cm/s (except in the simulation whose results are shown in Fig.2) due to the limitation of the embedded robot CPU (133MHz with 32MB RAM), which was capable to provide a maximum sampling rate of 4Hz for the addressed tasks.

Figure 2: Error vector and control signals for a rough transition.

The Figs. 2-4, 6, 7 are colored with black, blue and red, which mean the current visual servoing task (hallway following, transition, and positioning, respectively) over time. Also, the figures comprise, from left to right and from up to bottom, the error vectors of the line parameters \( s_1 - s_{1d} \), and of the normalized centroid image coordinates \( s_{2d} - s_{2d} \) for the 4 image points.

One can see in Fig. 2 the behavior of the system under a rough transition, where huge accelerations are demanded from the actuators, what could clearly cause damages. Those problems can be avoided by through a smooth transition, as shown in Fig. 3.

It is presented in Fig. 5 the navigation of the robot, as viewed by the aboard camera, during the switching between the corridor following and the positioning tasks. Since it was given a desired vector \( s_{1d} = [\theta_{d}, \rho_{d}]^{T} = [26^\circ, 0]^{T} \), the robot Nomad 200 follows on the left side of the hallway. The corresponding visual and the actuators signals can be observed in Fig. 4, where it is used a polynomial transition function.

Using the intermediate phase methodology, it was considered a stabilizing task error parametrized in the image plane such that the control signal \( u_{q} = [v, \theta]^{T} \) in Eqs. (18) and (19) was given as

\[
u_{q} = \left[ -\beta \sum_{i=1}^{2} \left| y_i - y_{i|d} \right| + \left| x_i - x_{i|d} \right| \right] e_{s_{1d}},
\]

(22)

and whose simulation results can be seen in Fig. 6, for a rough transition, and in Fig. 7 for a polynomial transition function.
Figure 3: Error vector and control signals using a polynomial transition function.

Figure 4: Experimental results: error vector and control signals using a polynomial transition function.

Figure 5: From left to right and from up to bottom, evolution of the image plane during Robot navigation (corridor following then positioning).
6 CONCLUSIONS

This work presented two different methodologies to switch between two primary visual servoing tasks, in order to avoid possible system failures. Sufficient conditions for the stability were also discussed for the strategies. The approaches were validated both in simulation and experimentally using a mobile ground robot with an uncalibrated camera mounted in an eye-in-hand configuration. Future works in this direction include also their implementation in different robotic platforms, such as the aerial robotic airship of the project AURORA (Bueno et al., 2002), to perform the switching between line following (Silveira et al., 2003) and hovering tasks (Azinheira et al., 2002).

REFERENCES


